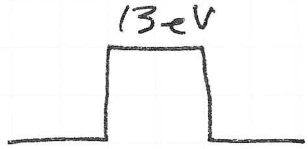
Physics 187 chapter #35
Homework Solutions

Problem #1 (P15): 10 eV electron



$a = 2.0\,nm.$

$$T = e^{-2\alpha a}, \quad a\alpha >> 1$$

(a) $$\alpha = \sqrt{\frac{2m(U_0 - E)}{\hbar^2}} = \frac{\sqrt{2m(U_0 - E)}}{\hbar}$$ multiply by $\frac{c}{c}$

$$\alpha = \frac{\sqrt{2mc^2(U_0 - E)}}{\hbar c}, \quad \hbar c = \frac{hc}{2\pi} = \frac{1240\,eV\cdot nm}{2\pi} = 197.4\,eV\cdot nm$$

$$mc^2 = .511 \times 10^6\,eV$$

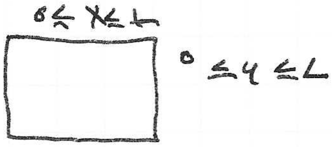
$$\alpha = \frac{\sqrt{2(.511 \times 10^6\,eV)(13\,eV - 10\,eV)}}{197.4\,eV\cdot nm} = 8.87$$

$$T = e^{-2(8.87)(2\,nm)} \qquad \boxed{T = 3.9 \times 10^{-16}}$$

(b) $a = 0.20\,nm$

$$T = e^{-2(8.87)(0.20)} =$$

Problem #2 (P21):



for x: $\sin \frac{n_1 \pi x}{L}$

for y: $\sin \frac{n_2 \pi y}{L}$

(a) $$\psi(x,y) = A \sin\left(\frac{n_1 \pi x}{L}\right) \sin\left(\frac{n_2 \pi y}{L}\right)$$

(b) Plug $\psi(x,y)$ into $-\frac{\hbar^2}{2m}\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2}\right) - U\psi(x,y) = E\psi(x,y)$

and Solve for E: $U = 0$

$$\frac{\partial \psi}{\partial x} = A\left[\sin\frac{n_2 \pi y}{L}\left(\frac{n_1 \pi}{L} \cos\frac{n_1 \pi x}{L}\right)\right] \qquad \frac{\partial^2 \psi}{\partial x^2} = -A \sin\frac{n_2 \pi y}{L}\left(\frac{n_1 \pi}{L}\right)^2 \sin\frac{n_1 \pi x}{L}$$

$$\frac{\partial^2 \psi}{\partial y^2} = -A \sin\frac{n_1 \pi x}{L}\left(\frac{n_2 \pi}{L}\right)^2 \sin\frac{n_2 \pi y}{L}$$

$$-\frac{\hbar^2}{2m}\left[-A \sin\frac{n_2 \pi y}{L}\left(\frac{n_1 \pi}{L}\right)^2 \sin\frac{n_1 \pi x}{L} - A \sin\frac{n_1 \pi x}{L}\left(\frac{n_2 \pi}{L}\right)^2 \sin\frac{n_2 \pi y}{L}\right] = E\psi$$

$$\frac{\hbar^2}{2m}\left[\left(\frac{n_1 \pi}{L}\right)^2 + \left(\frac{n_2 \pi}{L}\right)^2\right] = E \quad E = \frac{\hbar^2 \pi^2}{2mL^2}(n_1^2 + n_2^2), \quad \hbar = \frac{h}{2\pi}$$

$$\boxed{E_n = \frac{h^2}{8mL^2}(n_1^2 + n_2^2)}$$

Problem #2 (Cont):

(c) $\boxed{\psi_{12} \text{ and } \psi_{21}}$ have the same energy

(d) Three lowest states that have the same energy

$$E_n = \frac{h^2}{8mL^2}(n_1^2+n_2^2)$$

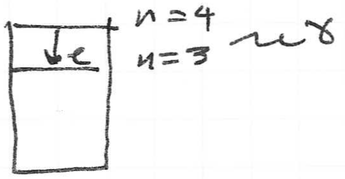
$n_1 = 1$, $n_2 = 7$	$E_n = 50$
$n_1 = 7$, $n_2 = 1$	$E_n = 50$
$n_1 = 5$, $n_2 = 5$	$E_n = 50$

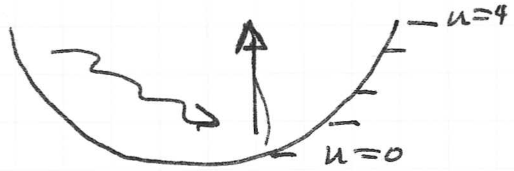
$\boxed{\text{Problem \#3 (P23)}}$: Ground state energy for 55 non-interacting bosons in a 1-dim box of length $= L$.

Each boson will have the same energy

$$\cancel{E_{70}}\,E_{55} = \cancel{70}\,55\,E_1 = \cancel{70}\,55\left(\frac{h^2}{8mL^2}\right) = \boxed{\frac{55}{\cancel{4}}\left(\frac{h^2}{8mL^2}\right)}$$

$\boxed{\text{Problem \#4 (PIE001)}}$





Determine ground state energy for Harmonic Oscillator:

$$E_{photon} = \Delta n^2 E_1 = (n_4^2 - n_3^2)\,\frac{\cancel{8m}h^2}{8mL^2}$$

$$E_{photon} = (16-9)\frac{(hc)^2}{8mc^2 (.4\,\cancel{\ }\,nm)^2} = \cancel{131.6\,eV}\ \cancel{1.5\,MeV}\ \cancel{131.6\,eV}\ 16.456\,eV$$

For a harmonic oscillator: $\cancel{E_n = (n+\tfrac{1}{2})\hbar\omega}$

$\cancel{E_4 - E_0 = \tfrac{5}{2}\hbar\omega - \tfrac{1}{2}\hbar\omega = 131.6\,eV}$

$\cancel{2\hbar\omega = 131.6\,eV}$

From interactive questions: $E_{ground\,HO} = \frac{1}{2}\Delta E_{spacing}$

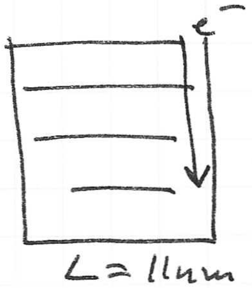
$E_{photon\,IW} = 4\,\Delta E_{spacing}$

$\Delta E_{spacing} = 2\,E_{ground\,HO}$, $\Delta E_{spacing} = \frac{1}{4}E_{photon\,IW}$

$2\,E_{ground\,HO} = \frac{1}{4}E_{photon\,IW}$, $E_{ground} = \frac{1}{8}E_{photon\,IW}$

$E_{photon\,IW} = 16.456\,eV \Longrightarrow \boxed{E_{ground\,HO} = 2.06\,eV}$

Problem #5 (IE002):



γ emitted = 875.53nm

$$E_\gamma = \frac{hc}{\lambda} = \frac{1240\,eV\cdot nm}{875.53\,nm}$$

$$E_\gamma = 1.42eV$$

$$\Delta E \text{ for transition} = (n+4)^2\frac{h^2}{8mL^2} - (n^2)\frac{h^2}{8mL^2}$$

$$\Delta E = [(n^2+8n+16) - n^2]\frac{h^2}{8mL^2} = 1.42eV$$

Solving for n: $$n = \frac{\frac{(1.42eV)8mL^2}{h^2} - 16}{8}$$

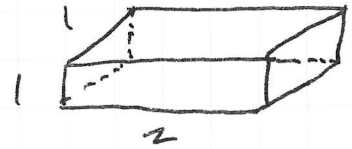
$n = 59.5$ $\quad n - 4 = \cancel{55.5}\ 55$

$n = 59$

$$E_{55} = n^2\frac{h^2}{8mL^2} = (55)^2\frac{(hc)^2}{8(mc^2)L^2} = \boxed{9.57eV}$$

$$\boxed{\text{final electron energy} = 9.57eV}$$

Problem #6 (IE003):



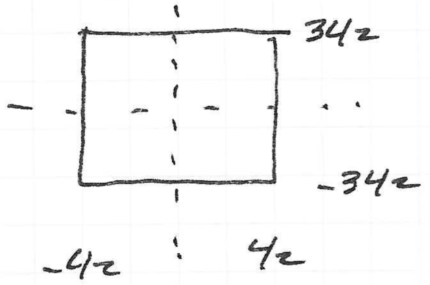
$x = 2.0$
$y = 1.0$
$z = 1.0$

$$\cancel{\psi(x,y,z) =}\ E(n_x,n_y,n_z) = \frac{h^2}{8m}\left[\frac{n_x^2}{a^2} + \frac{n_y^2}{b^2} + \frac{n_z^2}{c^2}\right].$$

$$E(n_x,n_y,n_z) = \frac{h^2}{8mL^2}\left[\frac{n_x^2}{4} + n_y^2 + n_z^2\right] = \frac{h^2}{32mL^2}\left(n_x^2 + 4n_y^2 + 4n_z^2\right)$$

$E(1,1,1)$	=	.846eV
$E(2,1,1)$	=	1.94 eV
$E(1,2,1)$	=	$E(1,1,2)$ = 1.97eV * 4Th Lowest.
$E(3,1,1)$	=	1.598eV

Problem#7 (P 30)



$u(x,y)=0$ within 2-d box

$u(x,y)=\infty$ elsewhere

(a)

$$E_n = \frac{h^2}{8m}\left(\frac{n_x^2}{L_1^2}+\frac{n_y^2}{L_2^2}\right)$$ let $L_1=L$, $L_2=3L$

$$E_n = \frac{h^2}{8m}\left(\frac{n_x^2}{L^2}+\frac{n_y^2}{9L^2}\right) = \frac{h^2}{72mL^2}\left(9n_x^2+n_y^2\right)$$

$$E_{1,1} = \frac{10h^2}{72mL^2} = \left(\frac{5}{36}\right)\frac{h^2}{mL^2}$$

$$E_{1,2} = \left(\frac{13}{72}\right)\frac{h^2}{mL^2}$$

$$E_{1,3} = \left(\frac{18}{72}\right)\frac{h^2}{mL^2} = \left(\frac{1}{4}\right)\frac{h^2}{mL^2}$$

(b) all the energies are unique, thus non are degenerate.

(c). Two lowest degenerate energy states:

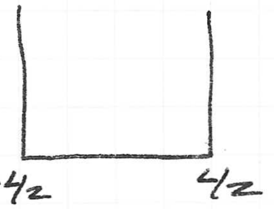
$$9n_1^2+n_2^2 = 9m_1^2+m_2^2$$ > let $n_1=1$, $m_1=2$

$$9+n_2^2 = 36+m_2^2$$ > $n_2=6$, $m_2=3$

$\boxed{\text{Thus } \psi(1,6), \psi(2,3) \text{ yield degenerate states}}$

$$\boxed{E = \frac{45}{72}\frac{h^2}{mL^2} = \frac{5}{8}\frac{h^2}{mL^2}}$$

Problem #8 (P 28):



(a) $\psi_m = \sqrt{\frac{2}{L}} \sin \frac{m\pi x}{L}$, $m = 2n$ $\psi_m = \sqrt{\frac{2}{L}} \cos \frac{m\pi x}{L}$, $m = 2n+1$

$n = 0, 1, 2, \ldots$

$\psi_1(x=0) \Rightarrow$ $\boxed{\psi_1(0) = 0, \quad \psi_2(0) = \sqrt{\frac{2}{L}}}$

Note: webassign has these reversed as input.

(b). $\langle x \rangle_{n=1} = \frac{2}{L}\int_{-L/2}^{L/2} x \sin^2 \frac{m\pi x}{L}\, dx = \frac{2}{L}\left[\frac{x^2}{4} - \frac{x \sin \frac{2m\pi x}{L}}{4\left(\frac{m\pi}{L}\right)} - \cos \frac{2m\pi x}{L}\right]_{-L/2}^{L/2}$

$$\boxed{\langle x \rangle_{n=1} = 0}$$

(c) for $n=1$ $\langle x^2 \rangle = \frac{2}{L}\int_{-L/2}^{L/2} x^2 \sin^2 \frac{\pi x}{L}\, dx = \boxed{\frac{L^2}{12}\left(1 + \frac{6}{\pi^2}\right)}$

Note: Webassign answer $\left(\frac{L^2}{12}\right)\left(1 - \frac{6}{\pi^2}\right)$

for $n=2$ $\langle x^2 \rangle = \frac{2}{L}\int_{-L/2}^{L/2} x^2 \sin^2 \frac{2\pi x}{L}\, dx = \boxed{\frac{L^2}{12}\left(1 + \frac{3}{2\pi^2}\right)}$

Note! Webassign answer $\left(\frac{L^2}{12}\right)\left(1 - \frac{3}{2\pi^2}\right)$

Problem #9 (P 16) $T = e^{-2\alpha a}$ $a = 3.0 \times 10^{-15}\text{ m} = .000003\text{ nm}$ ~~3000000nm~~

$\alpha = \sqrt{\frac{2m\,\Delta E}{\hbar^2}} = \frac{\sqrt{2m\,\Delta E}}{\hbar} = \frac{\sqrt{2mc^2\,\Delta E}}{\hbar c} = \frac{\sqrt{2mc^2\,\Delta E}}{\frac{1240\text{ eV nm}}{2\pi}}$

$\Delta E = 6\text{ MeV} = 6 \times 10^6\text{ eV}$

Determine mc^2 for a proton: $m = 1.6726 \times 10^{-27}\text{ kg}$

$mc^2 = (1.6726 \times 10^{-27}\text{ kg})(9 \times 10^{16}\text{ m/s}^2) =$ ~~1.505 x 10^{-10} Joules~~ $1.4 \times 10^{-10}\text{ J}$

$mc^2 = 939.7\text{ MeV}$

$\alpha = \frac{\sqrt{2(939.7\text{ MeV})(6 \times 10^6\text{ eV})}}{197.4\text{ eV}\cdot\text{nm}} = 5.38 \times 10^5$

$T = e^{-2(.000003)(5.38 \times 10^5)} = \boxed{.0396}$

Problem #10 (P22)

$\psi(x_1, x_2) = A\sin\left(\frac{\pi x_1}{L}\right)\sin\left(\frac{2\pi x_2}{L}\right)$

$-\frac{\hbar^2}{2m}\left[\frac{\partial^2\psi}{\partial x_1^2} + \frac{\partial^2\psi}{\partial x_2^2}\right] + U\psi = E\psi$

$\frac{\partial\psi}{\partial x_1} = A\left(\frac{\pi}{L}\right)\sin\frac{2\pi x_2}{L}\left(\cos\frac{\pi x_1}{L}\right) \qquad \frac{\partial^2\psi}{\partial x_1^2} = -A\left(\frac{\pi}{L}\right)^2\sin\frac{2\pi x_2}{L}\sin\frac{\pi x_1}{L}$

$\frac{\partial\psi}{\partial x_2} = A\left(\frac{2\pi}{L}\right)\sin\frac{\pi x_1}{L}\left(\cos\frac{2\pi x_2}{L}\right) \qquad \frac{\partial^2\psi}{\partial x_2^2} = -A\left(\frac{2\pi}{L}\right)^2\sin\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L}$

Substitute into Schrödinger Equation:

$-\frac{\hbar^2}{2m}\left[-A\left(\frac{\pi}{L}\right)^2\sin\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L} - A\left(\frac{2\pi}{L}\right)^2\sin\frac{\pi x_1}{L}\sin\frac{2\pi x_2}{L}\right] + 0\,\psi_{12} = E\,\psi_{12}$

$+\frac{\hbar^2}{2m}\left[\left(\frac{\pi}{L}\right)^2 + \left(\frac{2\pi}{L}\right)^2\right] = E$

$+\frac{\hbar^2}{2m}\left[\frac{\pi^2 + 4\pi^2}{L^2}\right] = E \quad > \quad \boxed{\frac{5\pi^2\hbar^2}{2mL^2} = E}^{*}, \quad \hbar = \frac{h}{2\pi}$

$E = \frac{5\pi^2 h^2}{2mL^2(2\pi)^2} = \boxed{\frac{5h^2}{8mL^2}}$

This proves required solution.

* is required answer for Webassign.