

TANGENTIAL and RADIAL VECTORS

Since all vectors have a magnitude and direction, it is possible for the vector to change in two distinct ways. The vector can change its magnitude and it can change its direction. The vector that describes how another is changing its magnitude is called a tangential vector because this second vector is directed tangential to the first vector's direction. If the first vector's magnitude is increasing, then the second vector points parallel to the first vector's direction. If the first vector is decreasing, the second vector points in the opposite direction.

The vector that describes how another vector is changing its direction is called a radial vector because the second vector is directed radially inward from the curve that describes the changing vector direction.

A good example is the velocity and acceleration vectors. You can change the magnitude of your car's velocity or speed by using the gas or brake pedals. This produces a tangential acceleration, a_t . You can change the direction of your car's velocity by using the steering wheel. This produces a radial or centripetal acceleration, a_r .

$a_t = dv/dt \hat{\theta}$, where v is the magnitude of \vec{v} and $\hat{\theta}$ is a unit vector whose direction is parallel to the trajectory but MAY NOT be constant. $\hat{\theta}$ will be constant for only a straight line trajectory.

$a_r = a_c = v^2/r (-\hat{r})$: where v is the instantaneous speed, r is the instantaneous radius of curvature, and \hat{r} is a unit vector that points radially outward from the center of curvature and is NEVER constant in direction. a_r always points inward or in the negative \hat{r} direction. Thus the centripetal acceleration vector is never constant.

The picture below has a car going around a circle while increasing its speed. This is not uniform circular motion because the speed is increasing. This is a special case where the radius of curvature is a constant. $\hat{\theta}$ is parallel to \vec{v} and perpendicular to both \hat{r} and \vec{a}_r .

