Cylindrical and Spherical Coordinate Systems

Cylindrical System:

The gradient operator in the cylindrical coordinates is

\[ \nabla_{\text{cyl}} = r \left( \frac{\partial}{\partial r} \right) + \theta \left( \frac{\partial}{\partial \theta} \right) + z \left( \frac{\partial}{\partial z} \right) \]

The line element in the cylindrical coordinates is

\[ dl = r \, dr + (\theta/r) \, d\theta + z \, dz \]

Note: \( r, \theta, z \) are the associated unit vectors
Spherical System:

The gradient operator in the spherical coordinates

\[ \nabla_{\text{sph}} = r \left( \frac{\partial}{\partial r} \right) + \phi \left( \frac{1}{r} \right) \left( \frac{\partial}{\partial \phi} \right) + \theta \left( \frac{1}{r \sin \phi} \right) \left( \frac{\partial}{\partial \theta} \right) \]

The line element in the spherical coordinates

\[ dl = r \, dr + r \, \phi \, d\phi + \theta \, r \, \sin \phi \, d\theta \]

*Note*: \( r, \theta, \phi \) are the associated unit vectors. Also note the change in the \( \theta \) and \( \phi \) directions for the spherical system, as compared with the cylindrical system.
Practice Problems
Cylindrical and Spherical Coordinate Systems.

Problem #1: Determine the potential difference in moving a 5-microcoulomb charge from the origin \((r, \theta, \phi) = (0,0,0)\) to a point \((2m, \pi/2, \pi/4)\) against the electric field \(E = -16r^2 r + (10/r \sin \phi) \theta \) (V/m).

Solution:

Problem #2: For a line charge with linear charge density \(\lambda = (10^{-9}/2) \) C/m on the z-axis, determine the potential difference \(V_{AB}\), where A is \((r, \theta, z) = (2m, \pi/2,0)\) and B is \((4m,\pi, 5m)\).

Solution:

Problem #3: Given the potential function \(V(r, \theta, z) = 6r^3 z^2 \cos \theta\), determine the \(E\) field at the point in space \((r, \theta, z) = (1m, \pi/6, 1m)\).

Solution:

Problem #4: A spherical conducting shell of radius \(a\), centered at the origin, has a potential field

\[
V = \begin{cases} 
V_0 \text{ for } r \leq a \\
\text{and} \\
\left(\frac{V_0 a}{r}ight) \text{ for } r > a 
\end{cases}
\]

Determine the electric field both inside and outside the shell.

Solution:
Additional information on coordinate systems may be found at:

http://mathworld.wolfram.com/topics/CoordinateGeometry.html