Key Pg#1

Physics 196
Chapter 21, 22, 23
Exam

Place all work on exam

ACADEMIC DISHONESTY WILL RESULT IN A ZERO GRADE FOR THE EXAM.

Conceptual Problems: **Do all 5 Problems. (2 points apiece)**

1. Rank the force experienced by charge q_2, given the second charge:
a). q_1 = 10micro coulombs at 1.4 meter
b). q_1 = 8 micro coulomb at 1.4 meters
c). q_1 = 4 micro coulombs at 1.4 meter

$\boxed{c, b, a}$

2. Rank the following by smallest to largest transfer of charge. Objects are the same size
$\Delta Q = 0$ **a) A pair of 1000 micro coulomb objects in contact**
$\Delta Q = 35$ **b) An 80 micro coulomb and 10 micro coulomb object in contact**
$\Delta Q = 50$ **c) A 200 micro coulomb and a 300 micro coulomb object in contact**

$\boxed{a, b, c}$

3. Rank the following for the potential at .5 meter from the charge
a). q_1 = 100 micro coulombs
b). q_1 = 10 micro coulomb
c). q_1 = 1 micro coulombs
d). q_1 = 1000 micro coulombs

$\boxed{c, b, a, d}$

4. 3 charges are individually positioned at 1 meter intervals from the previous charge. The total work done in establishing the charges depends on the order in which the charges are positioned:
(a).TRUE.
(b).FALSE.

FALSE

5. A charged particle moves upward in an electric field that point upward. The charge on the particle is:
(a) Negative
(b) Neutral
(c) Positive
(d) Cannot determine charge from information given

$\boxed{c}$ Positive charge moves in direction of $\vec{E}$ field.

Key Pg #2

Low Difficulty problems: Do all 4 Problems (10 points apiece):
All work must be shown for full credit.

A particle of charge -1.2milli coulombs and velocity (1.2 x 10^6 m/s) –i, enters an E field of strength and direction (2000N/C) j. If the mass of the charge is 4.5.x 10^{-16} kg, determine the acceleration and deflection direction of the particle.

$$a = \frac{F}{m} = \frac{Eq}{m} = \frac{(2000\frac{N}{C})(1.2\times10^{-3}C)}{4.5\times10^{-16}kg} = 5.33\times10^{15}\frac{m}{s^2}$$

Negative charges move in the opposite direction of the electric field. Thus the electron moves in the $-\hat{j}$ direction.

2. Determine the potential difference due to moving a 10 micro coulomb charge from 100 cm to a point 35 cm from a charge of 15 micro Coulombs.

$$\Delta V = \frac{kq}{r_f} - \frac{kq}{r_i} = kq\left(\frac{1}{r_f} - \frac{1}{r_i}\right) \qquad r_i = 1.00m \quad r_f = .35m$$

$$q = 15\times10^{-6}C.$$

$$\Delta V = \left(8.99\times10^9\ \frac{Nm^2}{C^2}\right)\left(15\times10^{-6}C\right)\left[\frac{1}{.35m} - \frac{1}{1.00m}\right]$$

$$\Delta V = 2.50\times10^5 \text{ Volts}$$

3. Calculate the potential difference in moving a charge +q from the point x = 1.0m to the point x = 2.5m against the electric field $E(x) = (x^4 + x + 10)\ i$

$$\Delta V = -\int_{1.00m}^{2.5m} \vec{E} \cdot \vec{dl} \qquad \vec{dl} = \hat{i}dx + \hat{j}dy + \hat{k}dz$$

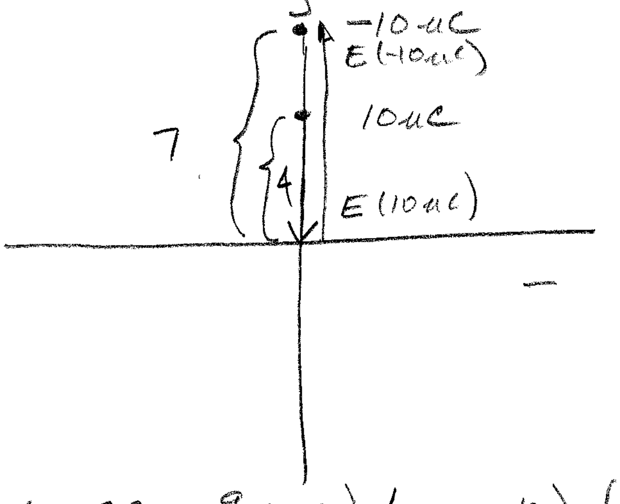
$$\Delta V = -\int_{1}^{2.5} (x^4 + x + 10)\hat{i} \cdot \hat{i}dx + \hat{j}dy + \hat{k}dz$$

$$\Delta V = -\int_{1}^{2.5} x^4 + x + 10\ dx = -\left[\frac{x^5}{5} + \frac{x^2}{2} + 10x\right]_1^{2.5}$$

$$\Delta V = -\left[\frac{(2.5)^5}{5} + \frac{(2.5)^2}{2} + 10(2.5) - \left(\frac{1}{5} + \frac{1}{2} + 10\right)\right]$$

$$\Delta V = -36.96 \text{ Volts.}$$

4. A 10 micro coulomb charge is situated at the coordinates x = 0 and y = 4.0 m. A second charge of -10 micro coulombs is situated at the coordinates x = 0 and y = 7.0 m. Determine the vector E field at the origin.



$$E_{TOTAL} = \Sigma E\,\hat{i}$$

$$\Sigma E\hat{i} = \frac{kq_1}{r_1^2} + \frac{kq_2}{r_2^2}$$

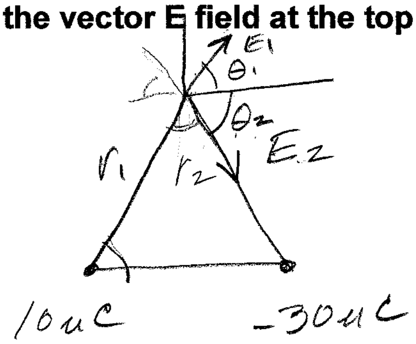
$$\Sigma E\hat{i} = \frac{\left(8.99\times10^9 \frac{Nm^2}{C^2}\right)(10\times10^{-6}C)(-\hat{j})}{(4m)^2} + \frac{8.99\times10^9 \frac{Nm^2}{C^2}(10\times10^{-6}C)\hat{j}}{(7m)^2}$$

$$\Sigma E\hat{i} = -3784\ \frac{N}{C} = 3784\ \frac{N}{C}(-\hat{j})$$

Key Pg #4

Advanced Difficulty Problems: **Do all 4 problems. 25 points apiece.**
All work must be shown for full credit!

1. A 10 micro-Coulomb charge and a -30 micro-Coulomb charge are placed at the bottom vertices of an equilateral triangle of length 1.0 meters. Determine the vector E field at the top vertex of the triangle.



$E_{TOTAL} = \sum E_i$
must resolve E_1 & E_2 into x & y components:

$$E_{1x} = \frac{kq_1}{r_1^2}\cos\theta_1 = \frac{8.99\times10^9 \frac{Nm^2}{C^2}\, 10\times10^{-6}C}{1m^2}\cos 60^\circ = 4.495\times10^4 \frac{N}{C}$$

$$E_{1y} = \frac{kq_1}{r_1^2}\sin\theta_1 = \frac{8.99\times10^9 \frac{Nm^2}{C^2}\, 10\times10^{-6}C}{1m^2}\sin 60^\circ = 7.79\times10^4 \frac{N}{C}$$

$$E_{2x} = \frac{kq_2}{r_2^2}\cos\theta_2 = \frac{8.99\times10^9 \frac{Nm^2}{C^2}\, 30\times10^{-6}C}{1m^2}\cos 60^\circ = 1.349\times10^5 \frac{N}{C}$$

$$E_{2y} = \frac{kq_2}{r_2^2}\sin\theta_2 = \frac{8.99\times10^9 \frac{Nm^2}{C^2}\, 30\times10^{-6}C}{1m^2}\sin 60^\circ = 2.33\times10^5 \frac{N}{C}$$

$E_{1x} = 4.495\times10^4 \frac{N}{C}\,\hat{i}$, $E_{1y} = 7.79\times10^4 \frac{N}{C}\,\hat{j}$

$E_{2x} = 1.349\times10^5 \frac{N}{C}\,\hat{i}$, $E_{2y} = 2.33\times10^5 \frac{N}{C}\,(-\hat{j})$

$$\sum E_x = 4.495\times10^4 \frac{N}{C} + 1.349\times10^5 \frac{N}{C} = 1.799\times10^5 \frac{N}{C}\,\hat{i}$$

$$\sum E_y = 7.79\times10^4 \frac{N}{C} - 2.33\times10^5 \frac{N}{C} = -1.55\times10^5 \frac{N}{C}\,(-\hat{j})$$

$$E_T = \sqrt{E_x^2 + E_y^2} = \sqrt{\left(1.799\times10^5 \tfrac{N}{C}\right)^2 + \left(-1.55\times10^5 \tfrac{N}{C}\right)^2} = 2.37\times10^5 \frac{N}{C}$$

$$\theta = \tan^{-1}\frac{E_y}{E_x} = \tan^{-1}\left(\frac{-1.55\times10^5}{1.799\times10^5}\right) = -40.75^\circ = 319^\circ$$

$\vec{E} = 2.37\times10^5 \frac{N}{C}$ @ 319° wrt x-axis.

2. Determine the potential involved in moving a point charge from the origin to the point (r,θ,φ) = (2.0m, π/3, π/6) for the electric field

$$E(r,\theta,\phi) = r\sin(\theta)\cos(\phi)\,\hat{r} + r\sin(\theta)\hat{\theta} + r\sin(\theta)\sin(\phi)\,\hat{\phi}$$

$\Delta V = -\int \vec{E}\cdot\vec{dl}$ using sherical $\overline{dl}$

$\overline{dl} = \hat{r}dr + \hat{\theta} r\sin\phi\, d\theta + \hat{\phi} r\, d\phi$

$\vec{E}\cdot\vec{dl} = (r\sin\theta\cos\phi\,\hat{r} + r\sin\theta\,\hat{\theta} + r\sin\theta\sin\phi\,\hat{\phi})\cdot(\hat{r}dr + \hat{\theta} r\sin\phi\, d\theta + \hat{\phi} r\, d\phi)$

$\overline{E}\cdot\overline{dl} = r\sin\theta\cos\phi\, dr + r^2\sin\theta\sin\phi\, d\theta + r^2\sin\theta\sin\phi\, d\phi$

$-\int \vec{E}\cdot\vec{dl} = -\left[\sin\theta\cos\phi \int r\,dr + r^2\sin\phi \int \sin\theta\, d\theta + r^2\sin\theta \int \sin\phi\, d\phi\right]$

$-\left[\sin\theta\cos\phi \frac{r^2}{2} + r^2\sin\phi(-\cos\theta) + r^2\sin\theta(-\cos\phi)\right]$

Now evaluate (r,θ,ϕ) at $(0,0,0)$ and $(2,\frac{\pi}{3},\frac{\pi}{6})$

values at 0 are all 0, Thus we only need to evaluate at $(2,\frac{\pi}{3},\frac{\pi}{6})$

$\Delta V = -\left[\sin(\frac{\pi}{3})\cos(\frac{\pi}{6})\,2 - 4\sin(\frac{\pi}{6})\cos(\frac{\pi}{3}) - 4\sin(\frac{\pi}{3})\cos(\frac{\pi}{6})\right]$

$\Delta V = -[-2.6 \text{ Volts}]$

$\Delta V = 2.6$ Volts.

3. Determine the electric field for the potential function

$$\mathbf{V(r,\theta,\phi) = r^4 \sin\theta \cos^3\phi}$$

$$\vec{E} = -\vec{\nabla}_{SPH} V$$

$$\vec{E} = -\left[\hat{r}\frac{\partial}{\partial r}(r^4\sin\theta\cos^3\phi) + \frac{\hat{\theta}}{r\sin\phi}\frac{\partial}{\partial\theta}(r^4\sin\theta\cos^3\phi) + \frac{\hat{\phi}}{r}\frac{\partial}{\partial\phi}(r^4\sin\theta\cos^3\phi)\right]$$

$$\vec{E} = -\left[4r^3\sin\theta\cos^3\phi\,\hat{r} + \frac{\cos^3\phi\, r^4}{r\sin\phi}(\cos\theta) + \frac{r^4\sin\theta}{r}\,3\cos^2\phi(-\sin\phi)\right]$$

$$\vec{E} = -\left[4r^3\sin\theta\cos^3\phi\,\hat{r} + \frac{r^3\cos^3\phi}{\sin\phi}\cos\theta - 3r^3\sin\theta\cos^2\phi\sin\phi\,\hat{\phi}\right]$$

$$\vec{E} = -\left[4r^3\sin\theta\cos^3\phi\,\hat{r} + \frac{r^3\cos^3\phi\cos\theta}{\sin\phi}\hat{\theta} - 3r^3\sin\theta\cos^2\phi\sin\phi\,\hat{\phi}\right]$$

4. A solid sphere has radius R and volume charge density $\rho(r) = 6r^{-4}$.
Determine the:
a). E field inside the sphere.
b).The total charge contained within the sphere.
c). The E field outside the sphere.

a). Using Gauss' Law: $E_{in}\, 4\pi r^2 = \dfrac{Q_{enc}}{\epsilon_0}$

$$dq_{enc} = \rho(r)\,dV = \rho(r)\,4\pi r^2\,dr$$

$$Q_{enc} = \int dq_{enc} = \int 6r^{-4}\, 4\pi r^2\,dr = \int 6(4\pi r^{-2})\,dr$$

$$Q_{enc} = \int dq_{enc} = 24\pi \int r'^{-2}\,dr' = 24\pi\left(-\frac{1}{r'}\Big|_0^r\right)$$

ignore infinity at r'=0

charge is negative

$$Q_{enc} = \frac{-24\pi}{r}$$

$$E_{in}\,(4\pi r^2) = \frac{-24\pi}{\epsilon_0 r}$$

$$E_{in} = \frac{-24\pi}{4\pi\epsilon_0 r^3} = \frac{-6}{\epsilon_0 r^3}$$, E points inward

this is not necessary for problem

b). $$Q_{TOTAL} = \int_0^R \rho(r)\,dV = \int_0^R 6r^{-4}\,4\pi r^2\,dr$$

$$Q_{TOTAL} = 24\pi \int_0^R r^{-2}\,dr = \frac{-24\pi}{R}$$ ignore infinity at r=0

c) $$E_{out}\,4\pi r^2 = \frac{Q_{enc}}{\epsilon_0} = \frac{-24\pi}{R\epsilon_0}$$

$$E_{out} = \frac{-24\pi}{4\pi r^2 \epsilon_0 R} = -\frac{6}{\epsilon_0 R}\left(\frac{1}{r^2}\right).$$ E points inward.