Ch8: Systems of Particles

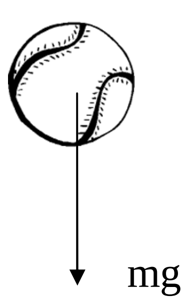
Center of Mass (CM) ⊗:

We wish to show that a body or system of particles move as if

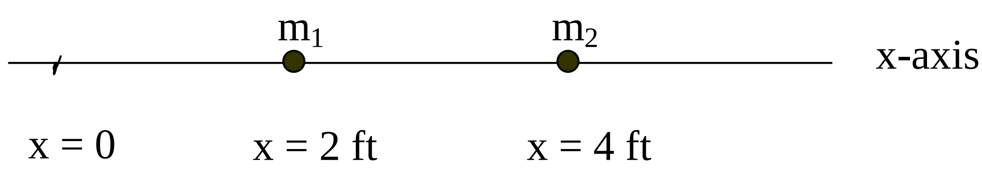
1. all of its mass were concentrated at one geometric point in space called the CM, and

2. the net external Force was applied there.

How many particles are in this system?



Consider a system of two point-particles.



$$x_{cm} \equiv \frac{m_1\, x_1 + m_2\, x_2}{m_1 + m_2}$$

This gives us the average position of the particles, weighted by their mass. (It's a weighted average).

If $m_1 = m_2 = m$

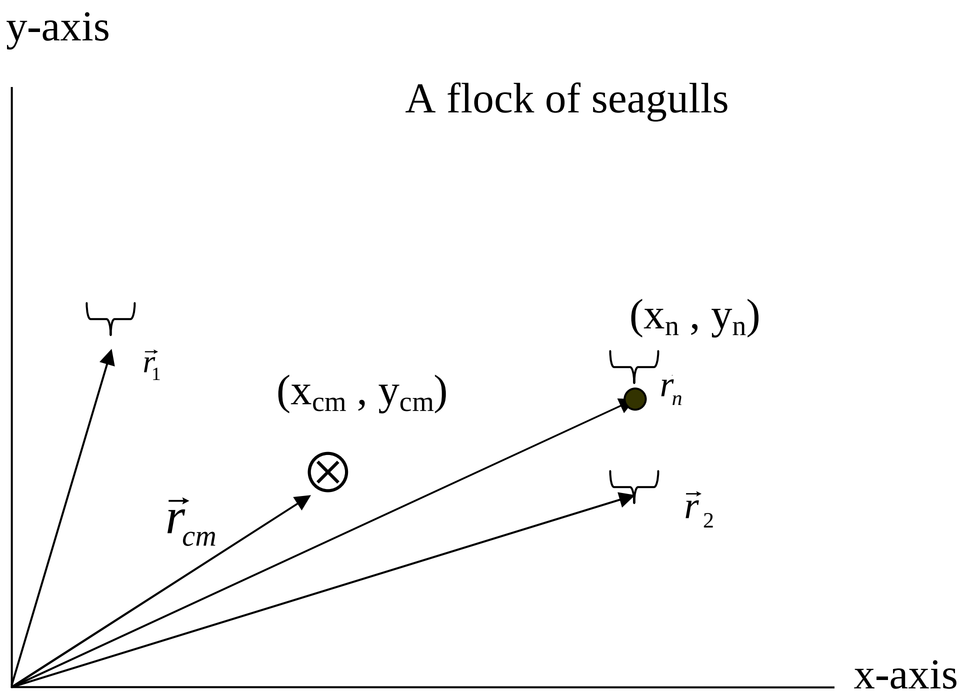
Then $x_{cm} = \dfrac{m(2\,ft) + m\;(4\,ft)}{m + m} = 3\,ft$, just as expected.

So, for n-particles (discrete masses)

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + ... + m_n x_n}{m_1 + m_2 + m_3 + ... + m_n} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i}$$

and

$$y_{cm} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + ... + m_n y_n}{m_1 + m_2 + m_3 + ... + m_n} = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i}$$



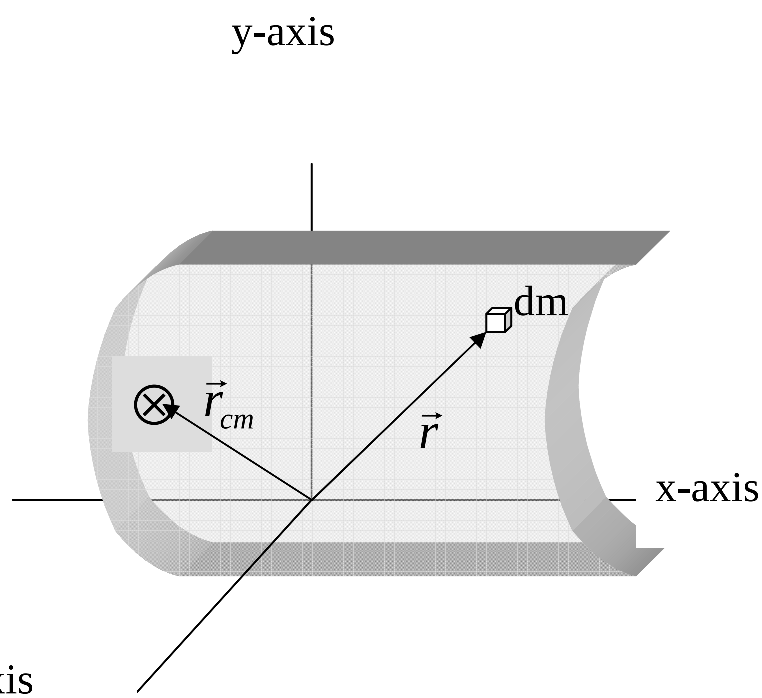
$\vec{r}_{cm}$ = the position of the CM of the system.

$$\vec{r}_{cm} = \frac{\sum_{i=1}^{n} m_i\, x_i}{\sum_{i=1}^{n} m_i}\,\hat{i} + \frac{\sum_{i=1}^{n} m_i\, y_i}{\sum_{i=1}^{n} m_i}\,\hat{j}$$

$$\vec{r}_{cm} = \frac{1}{\sum_{i=1}^{n} m_i}\left(\sum_{i=1}^{n} m_i \underbrace{\left(x_i\,\hat{i} + y_i\,\hat{j}\right)}_{\vec{r}_i}\right)$$

$$\therefore \quad \vec{r}_{cm} = \frac{\sum_{i=1}^{n} m_i \vec{r}_i}{\sum_{i=1}^{n} m_i}$$ for discrete masses (point-particles)

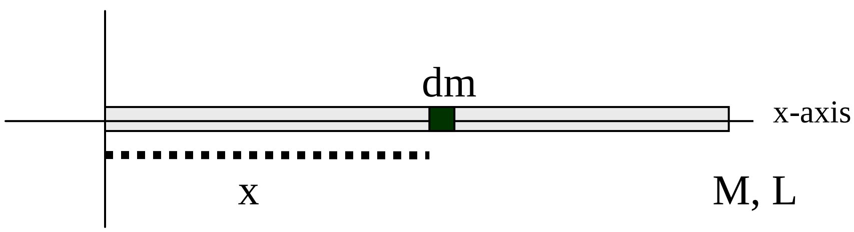
For a continuous distribution:



$$\vec{r}_{cm} = \frac{\int \vec{r}\, dm}{\int dm}$$

Ex: (1-dimension) Find the position of the center of mass of a thin homogeneous rod of length L and mass M. (ex: a meter stick)

$$x_{cm} = \frac{\int x\, dm}{\int dm}$$



the linear mass density λ is:

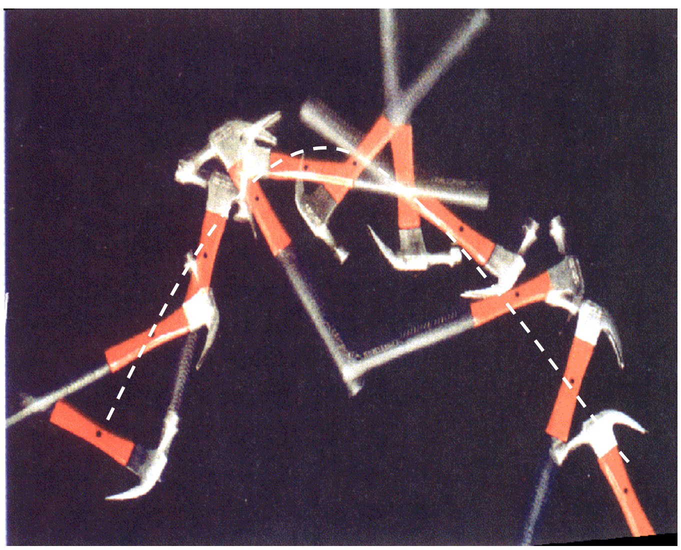
$$\lambda = \frac{mass}{length} \qquad \lambda = \frac{m}{x} \qquad m = \lambda x$$

so dm = λ dx

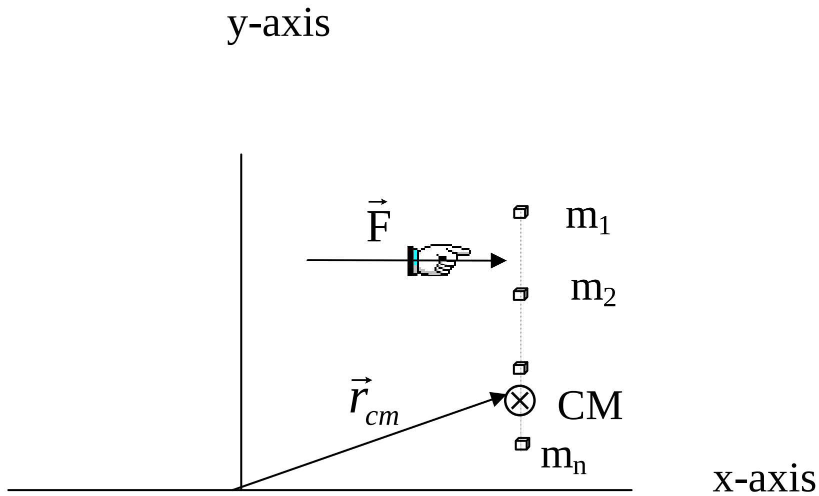
$$x_{cm} = \frac{\int x \lambda\, dx}{\int \lambda\, dx}$$ (limits?)

We can also find an objects CM by hanging it from a string.

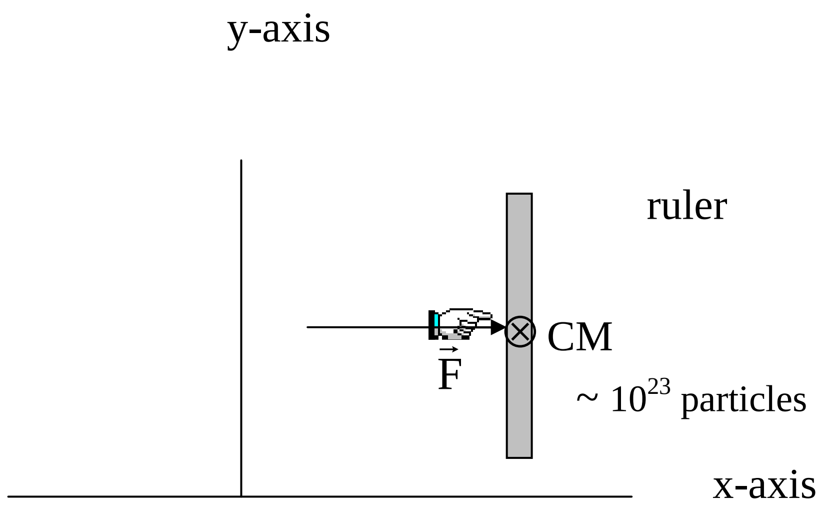
We can show that the CM of a body moves according to Newton's 2nd Law:



To do this, consider a system of n-particles (connected by tiny, massless, rigid rods) on which a quick force $\vec{F}$ acts.



How would this system of particles move? What if *n* was large?



Is Σ F = ma ?   Which m?   Which a?