$$\frac{d^2x}{dt^2} = -\sqrt{\frac{k}{m}}\sqrt{\frac{k}{m}}\, x_m \cos\left(\sqrt{\frac{k}{m}}\, t + \phi\right)$$

yields $$\frac{d^2x}{dt^2} = -\frac{k}{m}\underbrace{x_m \cos\left(\sqrt{\frac{k}{m}}\, t + \phi\right)}_{x}$$

so $$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$ ∴ we guessed a correct solution! ☺

when we had $\frac{d^2x}{dt^2} = -\frac{k}{m}x$, look at the units of

$$\frac{k}{m} : \frac{\left[\frac{N}{m}\right]}{[kg]} \frac{kg\, m}{N\, s^2} \Rightarrow \left(\frac{1}{s}\right)^2$$

or $\left(\frac{rad}{s}\right)^2$

∴ Let $\frac{k}{m} = \omega^2$

so $\frac{d^2x}{dt^2} = -\omega^2 x$ has the solution

$$x(t) = x_m \cos(\omega t + \phi), \quad \text{where } \omega = \sqrt{\frac{k}{m}}$$

$\omega \equiv$ the angular frequency