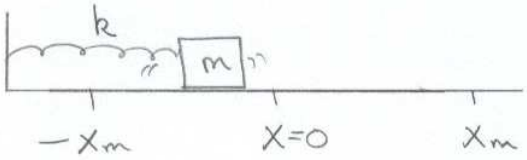
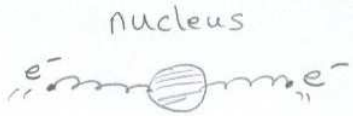
Here's a useful physical model:

Mass-spring system





Find $x(t)$ for this oscillating system.

$\sum F_x = ma_x$ but $F_s = -kx$

$-kx = ma \Rightarrow a = -\frac{k}{m}x$

* This is a property of SHM: $a \propto -x$

using $a = \frac{d^2x}{dt^2}$

$\frac{d^2x}{dt^2} = -\frac{k}{m}x$ 2nd order, linear, ordinary differential equation.

We'll Guess a solution out of thin air:

maybe $x(t) = x_m \cos\left(\sqrt{\frac{k}{m}}\, t + \phi\right)$, ϕ = constant

We must substitute this $x(t)$ back into our diff-eq to verify our guess:

$\frac{dx}{dt} = -\sqrt{\frac{k}{m}}\, x_m \sin\left(\sqrt{\frac{k}{m}}\, t + \phi\right)$ →