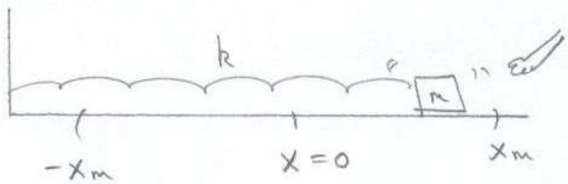
ENERGY of a SHO



Recall

$x = x_m \cos\omega t$

$v = -\omega x_m \sin\omega t$

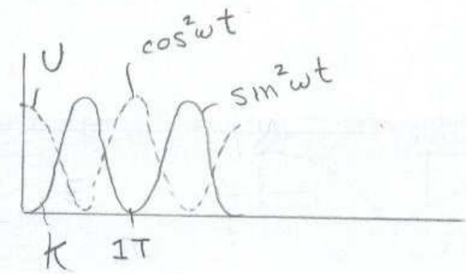
$K = \frac{1}{2}mv^2 = \frac{1}{2}m(\omega^2 x_m^2 \sin^2\omega t)$, but $\omega^2 = \frac{k}{m}$, $m\omega^2 = k$

$K = \frac{1}{2}k x_m^2 \sin^2\omega t$

$U = \frac{1}{2}k x^2$

$U = \frac{1}{2}k x_m^2 \cos^2\omega t$

U & K are out of phase (out of STEP) by $\frac{\pi}{4}$, or $\frac{1}{4}$ cycle.



The TOTAL Energy E is:

$E = K + U = \frac{1}{2}k x_m^2 \sin^2\omega t + \frac{1}{2}k x_m^2 \cos^2\omega t$

$E = \frac{1}{2}k x_m^2$ = constant

So,... where is $K = U$?

Set $K = U$ $\quad \frac{1}{2}k x_m^2 \sin^2\omega t = \frac{1}{2}k x_m^2 \cos^2\omega t \Rightarrow \sin^2\omega t = \cos^2\omega t$

$\omega t = \pm\frac{\pi}{4}$. $\quad x = x_m \cos\omega t$. $\quad$ when $\omega t = \frac{\pi}{4}$

$x = x_m \cos\frac{\pi}{4} = x_m \frac{1}{\sqrt{2}} \Rightarrow \therefore K = U$ when $x = \frac{x_m}{\sqrt{2}} \approx 70.7\% x_m$