Turbulent Dispersion in Stably Stratified Homogeneous Shear Flow

L. Pham, K. K. Nomura†, J. W. Rottman
Mechanical and Aerospace Engineering, University of California, San Diego,
La Jolla, CA 92093-0411
† email: knomura@ucsd.edu, phone: 858-534-5520, fax: 858-534-7599

Submitted to the 2005 Turbulence and Shear Flow Phenomena Symposium

Abstract

An important property of turbulence is its ability to disperse contaminants and mix scalar quantities. A primary application is the dispersion of pollutants in the atmosphere. Because of its vital importance, there have been significant efforts to develop models of turbulent dispersion. However, many uncertainties remain, in particular, in how to model dispersion in the stable boundary layer where both stratification and shear may be present.

In this work, direct numerical simulations (DNS) are performed to study the effects of stratification and shear on turbulent dispersion. The flow considered is homogeneous turbulence with uniform mean vertical shear and uniform stable stratification, the simplest flow in which both effects are present. Lagrangian statistics are obtained using particle tracking.

The three-dimensional, time-dependent continuity and Navier-Stokes equations with the Boussinesq approximation are solved using the numerical solution scheme of Gerz et. al. [3]. Periodic boundary conditions are employed in the x (streamwise) and y (spanwise) directions and shear-periodic conditions in the z (vertical/gravity) direction. The simulations are initialized with fully developed isotropic turbulent velocity field and zero scalar (density) fluctuations. Uniform mean vertical gradients of the velocity, $S \equiv dU/dz$, and density, $d\overline{\rho}/dz < 0$, are imposed. The initial Taylor microscale Reynolds number, $Re_\lambda = v\lambda/\nu$, is 30. The initial Shear number, $Sh = Sv^2/\varepsilon = 3.3$. The relative significance of stratification to mean shear effects is characterized by the gradient Richardson number, $Ri = N^2/S^2$, where $N = (-g\, d\overline{\rho}/dz/\,\rho_0)^{1/2}$ is the buoyancy frequency. A critical (stationary) value of Ri, $Ri_{cr} \approx 0.1$, designates the flow regimes in these flows [2]. A range of Ri corresponding to subcritical, critical, and supercritical flow conditions, which are associated with growing, stationary, and decaying turbulence, respectively, is considered.

Particle tracking is employed in order to obtain Lagrangian statistics. A cubic spline interpolation [9] is used to determine the fluid particle velocity from the grid point velocities. A set of 512 particles are uniformly distributed in the computational domain. However, displacement statistics are computed, through Galilean transformation, such that they effectively represent ensemble averages of particles which *originate* at the same location where the Eulerian mean velocity is zero. The particles are released (at time $St_o = 2$ in shear runs), after the turbulence has allowed to develop under the influence of the shear and stratification.

In neutrally-stratified, statistically stationary flow, fluid particles can move vertically over unlimited distances while the mean-square fluctuating velocity remains constant. As proposed by Taylor [8], at high Reynolds numbers, the effect of molecular diffusion on dispersion is negligible in comparison with convective transport. In stably-stratified stationary flow, in the absence of molecular diffusion, fluid particles are constrained to stay within a vertical distance of order w'/N from their equilibrium density level, where w' is the rms vertical component of turbulent velocity. However, molecular diffusion can alter the density of fluid particles which, in turn, alters the equilibrium level about which they oscillate with amplitude w'/N. Thus, the vertical flux of density in a stratified turbulent flow is the result of two processes: the first is the vertical displacements of fluid particles and the second is the molecular mixing between fluid particles. These ideas are the basis of the theory of Pearson, Puttock and Hunt [5], hereafter referred to as PPH, valid for statistically stationary homogeneous turbulence. In the present work, we test these ideas and investigate the effects of mean shear, which introduces additional dynamics and anisotropy into the flow.

Simulation results for unsheared flow ($Sh = 0$) are first considered. Figure 1 shows the time development of mean-square particle displacement in each of the coordinate directions, $\langle X^2 \rangle$, $\langle Y^2 \rangle$, $\langle Z^2 \rangle$. In the unstratified turbulence (Figure 1a), the three components remain equal since the flow

is isotropic. The short time behavior, $\langle Z^2 \rangle \sim t^2$, is consistent with theory [8], corresponding to nearly constant velocity and straight line motion. For long time, the theory of Taylor [8] predicts $\langle Z^2 \rangle \sim t$. The indicated slope of the curves at long time is slightly less than predicted by theory due to the nonstationarity of the flow which decays in time.

Figure 1b shows results for stratified unsheared flow ($N^2 = 1$). For short times, the results are in agreement with PPH which predicts the same behavior as in neutrally stratified flow, i.e., $\langle Z^2 \rangle \sim t^2$ [5], although $\langle Z^2 \rangle < \langle X^2 \rangle, \langle Y^2 \rangle$ due to suppressed vertical motion. For long times, the theory of PPH [5] predicts either $\langle Z^2 \rangle \sim (w/N)^2$ or $\langle Z^2 \rangle \sim t$, depending on the significance of molecular mixing in the flow. The present results indicate $\langle Z^2 \rangle$ terminates growth at $Nt \approx 3$ and then levels off. Results for a range of N^2 values (not shown) indicate that $\langle Z^2 \rangle$ levels off at progressively lower values (which scale with $1/N^2$) with increasing N^2. These results are consistent with results of previous DNS of stratified unsheared (decaying) homogeneous turbulence [4, 9]. Preliminary results indicate that most of the molecular mixing occurs at relatively early times in the flow and thus does not contribute to increasing dispersion at long times, at least for these decaying unsheared flows [9].

Figure 2a shows results for the unstratified ($Ri = 0$) shear flow. The behavior at short times is the same as that of the unsheared flow with $\langle X^2 \rangle$, $\langle Y^2 \rangle$, $\langle Z^2 \rangle \sim t^2$. For long times, the effects of shear are evident. The theory of Corrsin for stationary homogeneous shear flow [1] predicts $\langle X^2 \rangle \sim t^3$ and $\langle Y^2 \rangle$, $\langle Z^2 \rangle \sim t$. However, the DNS results exhibit slopes that are greater than those of the theory. This is consistent with previous DNS results [7, 6] and is attributed to the nonstationarity of the flow which exhibits growing turbulent velocities.

Figures 2(b)-(d) shows results for the stratified shear flows corresponding to subcritical ($Ri = 0.01$), near critical ($Ri = 0.1$), and supercritical ($Ri = 1.0$) flow conditions, respectively. The vertical displacement $\langle Z^2 \rangle$ decreases, with respect to $\langle Y^2 \rangle$, with increasing stratification. In general, the slopes of all three components of displacement are reduced. For the subcritical and critical flows (Figs. 2b,c), $\langle Z^2 \rangle$ does not level off to a constant value but continues to increase in time. The critical flow is of particular interest since it represents a (nearly) stationary flow. However, due to the effects of shear, $\langle Z^2 \rangle$ does not approach either of the limiting behaviors predicted by PPH. For the supercritical flow with $Ri = 1$, $\langle Z^2 \rangle$ levels off, as in the unsheared stratified flow (note that since $Ri = N^2/S^2 = 1$ (and $S = 1$), the level of stratification is the same in figures 1b and 2d).

Details of the analysis will be presented in the paper. Additional Lagrangian statistics are considered. The simulation results elucidate the effects of stratification and shear on vertical and horizontal displacements and the role of mixing.

References

[1] S. Corrsin. In *Proceedings of the Iowa Thermodynamics Symposium*, pages 5–30, University of Iowa, 1953.

[2] P. J. Diamessis and K. K. Nomura. The structure and dynamics of overturns in stably stratified homogeneous turbulence. *J. Fluid Mech.*, 499:197–229, 2004.

[3] T. Gerz, U. Schumann, and S. Elghobashi. Direct simulation of stably stratified homogeneous turbulent shear flows. *J. Fluid Mech.*, 200:563–594, 1989.

[4] Y. Kimura and J.R. Herring. Diffusion in stably stratified turbulence. *J. Fluid Mech.*, 328:253–269, 1996.

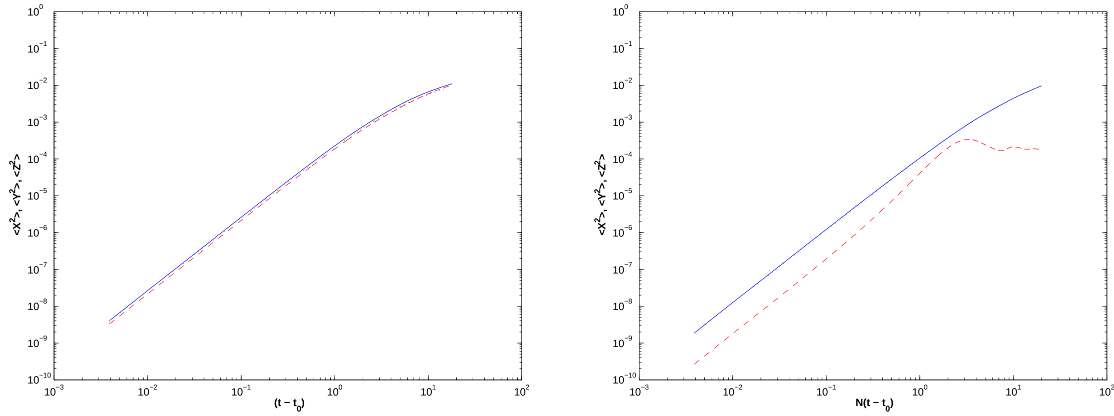
[5] H.J. Pearson, J.S. Puttock, and J.C.R. Hunt. A statistical model of fluid element motions and vertical diffusion in a homogeneous stratified turbulent flow. *J. Fluid Mech.*, 129:219–249, 1983.

[6] P. Shen and P. K. Yeung. Fluid particle dispersion in homogeneous turbulent shear flow. *Phys. Fluids*, 9:3472–3484, 1997.

[7] K. D. Squires and J. K. Eaton. Lagrangian and eulerian statistics obtained from direct numerical simulations of homogeneous turbulence. *Phys. Fluids*, 3:130–143, 1991.

[8] G.I. Taylor. Diffusion by continuous movements. *Proc. R. Soc. London Ser.*, A 20:196–211, 1921.

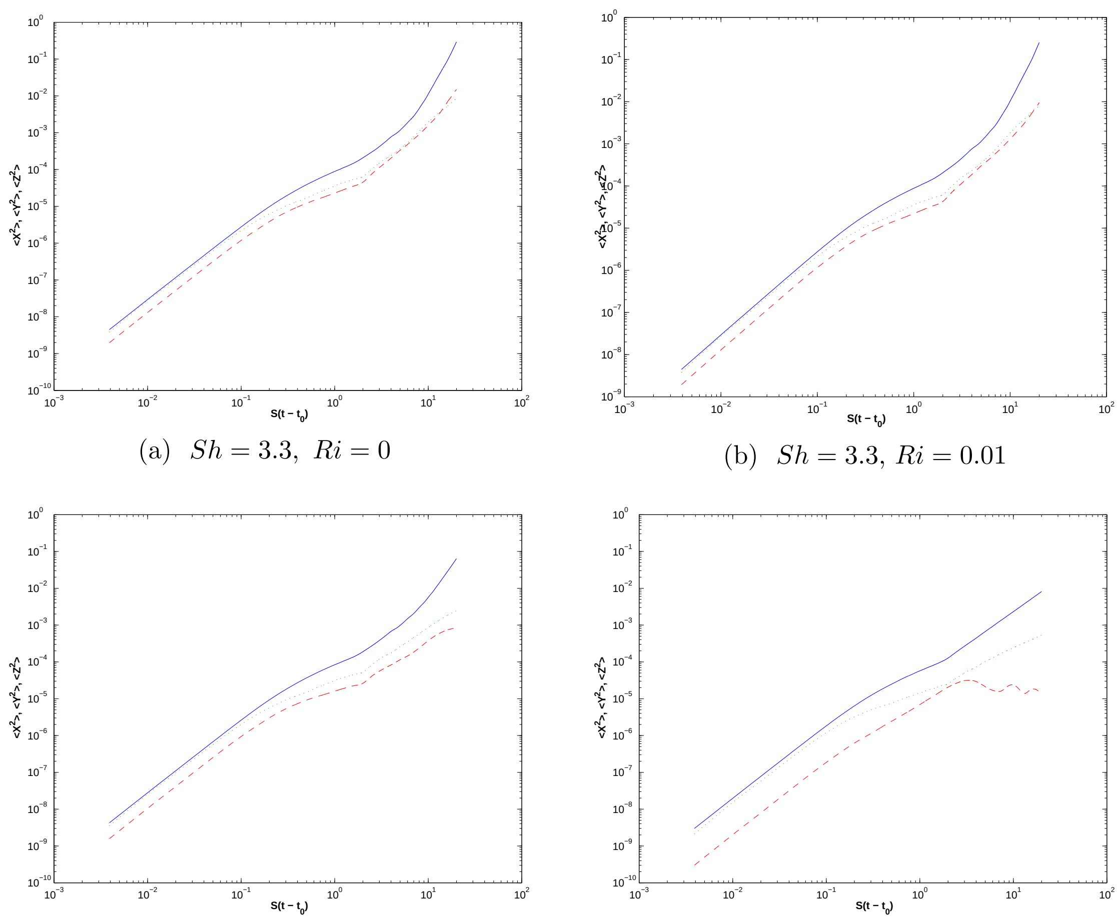
[9] S. K. Venayagamoorthy. Turbulent mixing and dispersion in environmental flows. Master's thesis, University of Natal, 2002.



(a) $N^2 = 0$

(b) $N^2 = 1$

Figure 1: Mean square particle displacement for unsheared flows (solid line: $\langle X^2 \rangle$, dotted line: $\langle Y^2 \rangle$, dash line: $\langle Z^2 \rangle$).



(a) $Sh = 3.3,\ Ri = 0$

(b) $Sh = 3.3,\ Ri = 0.01$

(c) $Sh = 3.3,\ Ri = 0.1$

(d) $Sh = 3.3,\ Ri = 1$

Figure 2: Mean square particle displacement for shear flows (solid line: $\langle X^2 \rangle$, dotted line: $\langle Y^2 \rangle$, dash line: $\langle Z^2 \rangle$).